

Baryons With Two Heavy Quarks as Solitons

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Abstract

Using the chiral soliton model and heavy quark symmetry we study baryons containing two heavy quarks. If there exists a stable (under strong interactions) meson consisting of two heavy quarks and two light ones, then we find that there always exists a state of this meson bound to a chiral soliton and to a chiral anti-soliton, corresponding to a two heavy quark baryon and a baryon containing two heavy anti-quarks and five light quarks, or a “heptaquark”.

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Recently a picture has emerged in which baryons containing a heavy quark, Q , and two light quarks, q 's, are viewed as bound states of a heavy meson made out of $Q\bar{q}$ in the field of a chiral soliton [1–4]. The heavy mesons are discussed using the Isgur-Wise heavy quark symmetry [5]; in this formalism all particles with spins made by combining a fixed spin for the light and for the heavy quarks are described by a single heavy field creating or annihilating particles of fixed four velocity. These ideas have been extended to bound states of a heavy anti-meson, $\bar{Q}q$ in the field of a chiral soliton [6]; this would lead to baryons (not anti-baryons) with charm $C = -1$ or beauty $B = +1$. In a conventional quark picture such states are made out of a heavy anti-quark and four light quarks and have previously been discussed [7] and named “pentaquarks”.

In this work we extend these ideas to baryons containing two heavy quarks. There are interesting differences from baryons with one heavy quark: in the latter case the ground state is, in the language of Ref. [1], in the $l = 1$ channel. For baryons with two heavy quarks the ground state consists of a superposition of $l = 0$ and $l = 2$ states (and possibly other states); independent of the sign of the potential, at least one state is bound for both the soliton and anti-soliton. The latter correspond in the conventional picture to states of two heavy anti-quarks and five light quarks or “heptaquarks”.

A crucial ingredient for the validity of this picture is the existence of stable, under strong interactions, bound states of $Q\bar{q} - Q\bar{q}$ mesons. A conservative estimate, using the one pion exchange potential between these particles, gives a binding for the B^*B system [8]. Bound states for the DD and BD systems are not excluded. It is likely that for sufficiently large m_Q/m_q these states will be bound in a configuration where the heavy and light systems are in antisymmetric color combinations [10]. For the case where there are stable $QQ\bar{q}\bar{q}$ systems we expect QQq baryons with spin-parity $\frac{1}{2}^+$; our results are consistent with this expectation and we likewise predict the existence of heptaquarks consisting of two \bar{Q} 's and five light quarks.

We analyze the $QQ\bar{q}\bar{q}$ system by first coupling the heavy quarks to each other, the light anti-quarks to each other and then combining the two. As mentioned earlier the favored

color configuration is for the QQ system to be in a color $\bar{\mathbf{3}}$ and the $\bar{q}q$ in a color $\mathbf{3}$. We shall also consider the possibility that they are in a symmetric color combination, namely $\mathbf{6}$ and $\bar{\mathbf{6}}$. First we shall look at the case where the two heavy quarks are identical. Then for the antisymmetric color combination the heavy quark spin $S_H = 1$ and we have the following possibilities:

$$\begin{aligned} (a) \quad & S_l = 1; \quad I = 1; \quad J = 0, 1, 2; \quad T_{iJ}^\alpha \\ (b) \quad & S_l = 0; \quad I = 0; \quad J = 1; \quad V_J \end{aligned} \tag{1}$$

For the color symmetric case, $S_H = 0$ and:

$$\begin{aligned} (a) \quad & S_l = 0; \quad I = 1; \quad J = 0; \quad S^\alpha \\ (b) \quad & S_l = 1; \quad I = 0; \quad J = 1; \quad V_i \end{aligned} \tag{2}$$

In the above S_l is the spin and I is the isospin of the light anti-quarks and J is the total spin of the $QQ\bar{q}q$ system. In the last column of the above equations we indicate the notation for heavy fields with three velocity zero that combine fields with fixed QQ and $\bar{q}q$ spin configurations. The lower case indices refer to light spin degrees of freedom and the upper case ones to the spin of the heavy quark combinations; isospin is indicated by upper Greek indices. The tensor fields T combine a spin zero, s , spin one, v_i and symmetric traceless spin two, t_{ij} , fields into a spin multiplet “superfield”

$$T_{iJ} = \frac{s}{\sqrt{3}}\delta_{iJ} + \frac{\epsilon_{iJk}v_k}{\sqrt{2}} + t_{iJ}. \tag{3}$$

Heavy quark spin symmetry demands invariance under rotations of the upper case spin indices.

With Σ , a unitary 2×2 matrix describing the light Goldstone pions and $\xi = \Sigma^{\frac{1}{2}}$, the Lagrangian for the heavy system (with zero three velocity) is

$$\begin{aligned} \mathcal{L} = & -iT_{iJ}^{\alpha\dagger}D_t^{\alpha\beta}T_{iJ}^\beta - iV_J^\dagger\partial_t V_J - iS^{\alpha\dagger}D_t^{\alpha\beta}S^\beta - iV_i^\dagger\partial_t V_i \\ & + g_1\epsilon^{\alpha\beta\gamma}\epsilon_{ijk}T_{iJ}^{\alpha\dagger}A_j^\beta T_{kJ}^\gamma + g_2(V_J^\dagger A_i^\alpha T_{iJ}^\alpha + \text{h.c.}) + g_3(S^{\alpha\dagger}A_i^\alpha V_i + \text{h.c.}). \end{aligned} \tag{4}$$

In the above

$$\begin{aligned}
D_t^{\alpha\beta} &= \delta^{\alpha\beta} \partial_t - \frac{1}{2} \epsilon^{\alpha\beta\gamma} \text{tr} \left[\tau^\gamma (\xi^\dagger \partial_t \xi + \xi \partial_t \xi^\dagger) \right] \\
A_i^\alpha &= \frac{i}{2} \text{tr} \left[\tau^\alpha (\xi^\dagger \partial_i \xi - \xi \partial_i \xi^\dagger) \right].
\end{aligned} \tag{5}$$

For completeness we also write down the version of the Lagrangian for arbitrary four-velocity v^μ of the heavy degrees of freedom as

$$\begin{aligned}
\mathcal{L}_V &= -i T^{\alpha\dagger \mu\nu} v \cdot D^{\alpha\beta} T_{\mu\nu}^\beta - i V^{\dagger \mu} v \cdot \partial V_\mu - i S^{\alpha\dagger} v \cdot D^{\alpha\beta} S^\beta - i V^{\dagger \mu} v \cdot \partial V_\mu \\
&+ g_1 \epsilon^{\alpha\beta\gamma} \epsilon^{\mu\nu\rho\sigma} T_{\mu\delta}^{\alpha\dagger} A_\nu^\beta T_{\rho\delta}^\gamma v_\sigma + g_2 (V^{\dagger \mu} A^{\alpha\nu} T_{\nu\mu}^\alpha + \text{h.c.}) + g_3 (S^{\alpha\dagger} A^{\alpha\mu} V_\mu + \text{h.c.}).
\end{aligned} \tag{6}$$

In the above α, β, γ are isospin indices and the field T is

$$\begin{aligned}
T_{\mu\nu} &= \frac{s}{\sqrt{3}} (-g_{\mu\nu} + v_\mu v_\nu) + \frac{\epsilon_{\mu\nu\rho\sigma} v_\rho v_\sigma}{\sqrt{2}} \\
&+ t_{\mu\nu} - t_{\mu\rho} v^\rho v_\nu - t_{\rho\nu} v^\rho v_\mu + t_{\rho\sigma} v^\rho v^\sigma v_\mu v_\nu,
\end{aligned} \tag{7}$$

and satisfies the constraints $v^\mu T_{\mu\nu} = v^\nu T_{\mu\nu} = 0$.

For the classical $SU(2) \times SU(2)$ soliton [11] $\Sigma(\vec{r}) = \exp[i\vec{\tau} \cdot \hat{r} F(r)]$, with $F(0) = -\pi$, we obtain

$$\begin{aligned}
A_i^\alpha(r) &= a_1(r) \delta_i^\alpha + a_2(r) \hat{r}_i \hat{r}^\alpha, \\
a_1(r) &= \frac{\sin F(r)}{2r}, \quad a_2(r) = \frac{r F'(r) - \sin F(r)}{2r}.
\end{aligned} \tag{8}$$

The intrinsic parity of the $QQ\bar{q}\bar{q}$ system is positive and as we expect the ground state of the baryon to be $\frac{1}{2}^+$ the $QQ\bar{q}\bar{q}$ must be in an even angular momentum state. For T_{ij}^α and for V_J , as defined in Eq. (1), we can do this with $l = 0$ and $l = 2$ waves.

$$\begin{aligned}
T_{ij}^\alpha &= (h_1(r) \delta_i^\alpha + h_2(r) \hat{r}_i \hat{r}^\alpha) \chi_J \\
V_J &= h_3(r) \chi_J;
\end{aligned} \tag{9}$$

χ_J is the wavefunction for the heavy spin, which decouples from the problem, as demanded by heavy spin symmetry. For the symmetric color configurations, Eq. (2), there are *no* even parity configurations possible. It is heartening that it is the color antisymmetric configurations of $QQ\bar{q}\bar{q}$ that are expected to be bound [10]. In the h_1, h_2, h_3 basis the potential energy matrix is

$$V = 2g_1 \begin{pmatrix} 2(3a_1 + a_2) & 2a_1 & \frac{g_2}{g_1}(3a_1 + a_2) \\ 2a_1 & 0 & \frac{g_2}{g_1}(a_1 + a_2) \\ \frac{g_2}{g_1}(3a_1 + a_2) & \frac{g_2}{g_1}(a_1 + a_2) & 0 \end{pmatrix}. \quad (10)$$

Let us first look for the eigenvalues of V for the case $g_2 = 0$. Two eigenvalues are of opposite sign and one is at zero; thus one of the configurations is bound. For small g_2/g_1 , independent of the sign of g_2 the level at zero repels the other two and one becomes more bound and the other more unbound. As can be seen in Fig. 1 and Fig. 2 this behavior persists at higher values of g_2/g_1 . We also note that the minimum eigenvalues occur at $r = 0$. We find that independent of the signs of the couplings we always have at least one bound state.

Now if we couple the $QQ\bar{q}\bar{q}$ system to the field of an anti-soliton, A_i^α changes sign and the positive eigenvalues of the soliton case become negative, resulting in heptaquark states. *This analysis predicts that a bound $QQ\bar{q}\bar{q}$ meson will yield at least one bound QQq and one $\bar{Q}\bar{Q}qqqqq$ baryon.* Zero mode quantization shows that it is the soliton that will determine the spin and isospin quantum numbers of these objects.

We briefly turn to the case when the two heavy quarks are not identical. The only change is that in both the color symmetric and color antisymmetric case the heavy quark spin can be both zero and one; there is no effect on the light quarks and thus the spectrum is doubled. This is consistent with what we would expect from the usual quark model description of these baryons.

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FIGURES

FIG. 1. Eigenvalues of the potential matrix for different values of g_2/g_1 . Solid line corresponds to $g_2/g_1 = 0$ and the dashed one to $g_2/g_1 = 1$.

FIG. 2. Same as Fig. 1 for $g_2/g_1 = 5$, solid line and $g_2/g_1 = 10$, dashed line.

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